

Partial Partial Preference Order Orders

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Consider the following collection of six-sided dice:

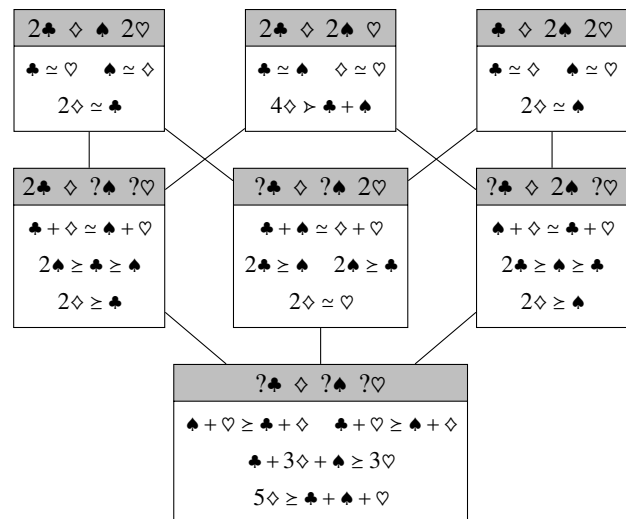
There are four faces, each present at least once: clubs ♣, spades ♠, diamonds ♦, and hearts ♥. A face only becomes visible after applying a drop of white wine to its side. There are at least three black faces. There are either more hearts than diamonds or an equal number of clubs and spades. A die is fair unless it has more black than white-faced sides, then each of the latter is equally more likely to land up than each of the former.

Because of the *exclusive disjunctions*—either/or statements—in this description, the uncertainty we must model when gambling with dice from this collection cannot be handled using a single convex credal set, set of desirable gambles, preference order, or other such uncertainty model. Arguably, also non-convex credal sets are inadequate here.

I wish to discuss the following conceptual approach for dealing with this modeling issue:

- The *possibility space* is restricted to observables only (♣, ♦, ♠, and ♥) and so should not involve, e.g., the die variant. (There are three such variants; see the gray boxes in the top row of the diagram.)
- We consider the *partial order* X generated by the exclusive disjunctions. (See the gray boxes and their interconnections in what is in fact a Hasse diagram.)
- We attach an *uncertainty model* to each element of X , e.g., a partial preference order, that reflects the information common to its upset in X . (In the diagram we use \geq for non-strict acceptance, \succ for strict preference, and \simeq for indifference [1]. Also, in the expressions, the faces denote the corresponding indicator gamble.)
- We can furthermore assign an *optimality criterion* to each element of X . Maximality and maximin variants thereof are natural candidates, E -admissibility perhaps less so, due to its use of individual probability measures, which can be replaced by exclusive disjunctions.

- With any set of decision options, we can then associate the corresponding partial order of optimal options. *Choice functions* [cf. 2] may be derived as functions thereof, for example the union of optimal options for the maximal elements of X .



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Keywords. Exclusive disjunction, partial order, uncertainty model, credal set, set of desirable gambles, preference order, optimality criterion, choice function.

References

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