Decision Theory Meets Linear Optimization Beyond Computation

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We consider the standard model of *finite* decision theory: An *actor* has to decide which *action* to pick from a finite set $\mathbb{A} = \{a_1, \dots, a_n\}$ of alternatives. However, the *utility* of the chosen action depends on which *state of nature* from a finite set $\Theta = \{\theta_1, \dots, \theta_m\}$ corresponds to the true description of reality. Specifically, we assume that the utility of every pair $(a, \theta) \in \mathbb{A} \times \Theta$ can be evaluated by a *known* map $u : \mathbb{A} \times \Theta \to \mathbb{R}$.

Within this framework, our goal is to determine an *optimal* action. However, any appropriate definition of optimality depends on (what we assume about) the *mechanism generating the states of nature*. Here, traditional decision theory mainly covers two *extremes*: The mechanism follows a *known* probability measure ξ on $(\Theta, \mathcal{P}(\Theta))$ or can be compared to a *game against an omniscient enemy*. Then optimality is almost unanimously defined by the *Bernoulli-criterion* (w.r.t. ξ) or the *Maximin-criterion*, respectively.

In contrast, defining optimality becomes less obvious if we consider ξ only *partially* known. Here, *imprecise probabilities* offer a powerful framework: Uncertainty is now described by the *credal set* of all the measures being compatible with our information (or by *linear partial information*, see [4]). However, criteria for optimal decision making now strongly depend on the actor's *attitude towards ambiguity*. Accordingly, many concurring decision criteria exist: Γ-*maximin*, Γ-*maximax, maximality, E-admissibility* [3, e.g.].

For determining optimal decisions w.r.t. these complex criteria *linear programming theory* (*LPT*) is well-suited: By embedding decision problems into this framework, one can draw on the whole toolbox of this well-investigated discipline. Particularly, this allows a computational treatment of complex decision making in statistical standard software (e.g. R): Proposals for linear programming based algorithms for optimizing all criteria mentioned above are given in [1] and [2].

However, the opportunities using *LPT* in decision theory are not exhausted by producing algorithms: Applying results from *LPT* provides deep theoretical insights on the connection between decision criteria as well as on the properties of optimal actions.

Firstly, we demonstrate the computational strength of *LPT* by recalling algorithms from [1] and [2] and exemplifying their implementation in R. Additionally, we introduce two algorithms for checking maximality of pure actions by solving *one single* linear program.

Secondly, we illuminate the power of *LPT* apart from algorithmic considerations: *Duality theory* from *LPT* is used to derive connections between optimal *randomized* Γ-maximin actions and *pure* Bernoulli-optimal actions w.r.t. a *least favourable* measure contained in the underlying credal set $\mathcal{M}$. We show that for every randomized $\Gamma(\mathcal{M})$-Maximin-optimal action p^*, there exists a pair $(a^*, \pi^*) \in \mathbb{A} \times \mathcal{M}$ such that $\mathbb{E}_{\pi^*}(u(a^*, \cdot))$ equals the $\Gamma(\mathcal{M})$-Maximin utility of p^*.

Keywords. decision making, imprecise probabilities, linear programming, partial information, ambiguity

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