Bayesian Nonparametric Tests Based on the Imprecise Dirichlet Process

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Empirical results of almost all scientific research are analyzed based on frequentist null hypothesis significance tests, even though the shortcomings of such methods are well known (consider, for instance, the recent decision of a psychology journal to ban null hypothesis significance tests from their articles [4]). This work on hypothesis testing based on the Imprecise Dirichlet Process (IDP) [2] aims to change this perspective by providing Bayesian versions of nonparametric frequentist tests.

The Imprecise Dirichlet Process The Dirichlet process (DP) is a natural prior for developing nonparametric tests in a Bayesian framework. It is completely defined by its prior strength s (a scalar) and its normalized base measure α (a probability measure). To overcome the problem of eliciting its infinite dimensional parameter α in case of lack of prior information, we have developed a prior near-ignorance DP model (IDP) that consists of the set of all DPs with fixed s and α free to vary in the set of all probability measures. Beside solving the prior elicitation problem, this model reduces the computational costs and provides posterior inference which are more robust with respect to the choice of the prior.

Nonparametric Hypothesis Tests Based on the IDP model we have developed imprecise Bayesian tests that share strong similarities with a number of frequentist statistics, and thus provide a Bayesian justification of many traditional nonparametric tests: the sign test [3], the Wilcoxon signed test [1], the Mann-Whitney-Wilcoxon rank-sum test [2] (including the case for censored data [5]), the Friedman test [1] and the Kendall tau test. In this Bayesian framework, tests are formulated as decision problems where the goal is to minimize the expected loss. Such a principled way of balancing significance and power of the test is lacking in the frequentist setting. Moreover, IDP based tests automatically inform the analyst when the decision minimizing the expected loss changes depending on the DP base measure. In these prior-dependent cases the test issues an indeterminate outcome. We have empirically verified that, often, traditional tests virtually behave as random guessers in these indeterminate instances.

Conclusions By making the elicitation of the DP prior easier, computations faster and posterior inferences more reliable, the IDP model allows performing simple and efficient nonparametric hypothesis tests in a Bayesian way. These tests have several advantages: they avoid the shortcomings of the frequentist ones, formulate the hypothesis test as a decision problem, are conservative with respect to the choice of the prior and automatically inform when the decision is difficult (and thus traditional tests are not reliable). Due to all these qualities, these test can challenge the widespread use of nonparametric frequentist test in all areas of scientific research.

Keywords. Dirichlet process, nonparametric hypothesis testing, prior near-ignorance.

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