An Idea of Consonant Conflicts between Belief Functions

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Introduction General belief functions usually bear some internal conflict, which comes mainly from disjoint focal elements. Analogously there is often some conflict between two (or more) belief functions (BFs). This theoretical contribution introduces a new approach to conflicts of BFs. Conflicts between BFs are here considered independently of any combination rule and of any distance measure.

Consonant Conflicts The suggested approach is based on consonant approximations of BFs in general; two important special cases based on consonant inverse pignistic and consonant inverse plausibility transformations are discussed. Their idea is based on our previous study of conflicts of BFs [1, 2, 3].

Probabilistic approximations of belief functions were used in several previous approaches, e.g. pignistic probability in W. Liu's two-dimensional degree of conflict and in pignistic conflict [2], and normalized plausibility of singletons in plausibility conflict [1, 2].

Unfortunately, doing a probability approximation usually adds new conflicting information, which increases internal conflict of input beliefs and also resulting global conflict. There are many inverses of any probabilistic approximation, in general (a mapping back to original input BFs among them), nevertheless, there are unique consonant inverses of both pignistic and plausibility probabilistic transformations. These inverses are internally non-conflicting (they have no internal conflict). Thus the entire global conflict of these approximations is the conflict between them (there is no conflict inside them). Our present idea is use of consonant instead of probabilistic approximations.

Definitions Let *the consonant inverse contour approximation* $iC(Bel)$ of a BF Bel be the unique consonant inverse of *the normalized plausibility of singletons (normalized contour function)* corresponding to Bel.

Let *the consonant inverse pignistic approximation* $iBet(Bel)$ of a BF Bel be the unique consonant inverse of *the pignistic probability* corresponding to Bel.

Let Bel_1, Bel_2 be any belief functions on any frame Ω, $iC(Bel_i)$ and $iBet(Bel_i)$ be their consonant inverse contour and consonant inverse pignistic approximations given by consonant bbas ${}_{iC}m_i$, ${}_{iBet}m_i$. *The inverse contour conflict* is defined by the formula $iC\text{-}Conf(Bel_1, Bel_2) = \sum_{X\cap Y=\emptyset} {}_{iC}m_1(X){}_{iC}m_2(Y)$, where $X, Y \subseteq \Omega$. *The inverse pignistic conflict* is analogously defined by $iBet\text{-}Conf(Bel_1, Bel_2) = \sum_{X\cap Y=\emptyset} {}_{iBet}m_1(X){}_{iBet}m_2(Y)$, where $X, Y \subseteq \Omega$.

Properties In [4] we have proved an equivalence of the consonant conflict $iC\text{-}Conf$ with the conflict between BFs based on their con-conflicting parts [3]. For quasi Bayesian BFs (focal elements: $|X| = 1$ or $X = \Omega$) Bel_1, Bel_2 with bbas m_1, m_2 we have proved: $Conf(Bel_1, Bel_2) \leq \sum_{X\cap Y=\emptyset} m_1(X)m_2(Y)$ for both $iC\text{-}Conf$ and $iBet\text{-}Conf$. Note that this does not hold for general BFs. For more detail, general counterexample, and other properties see [4].

Keywords. Belief functions, Dempster-Shafer theory, internal conflict of a belief function, conflict between belief functions, consonant approximation.

References

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