A Unified Model of Inductive Reasoning

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We offer a model that can capture three types of reasoning.[1] The first, which is the most common in economic modeling, is *Bayesian*. The agent formulates the set of possible states of the world and a prior probability distribution over this state space. The agent's predictions are a relatively straightforward matter of applying Bayes' rule, as new observations allow her to rule out some states and condition her probability distribution on the surviving states.

An alternative mode of reasoning is *case-based*. The agent considers past observations and predicts the outcome that appeared more often in those past cases that are considered similar. If all past observations are considered equally similar, the case-based prediction is simply the mode, that is, the outcome that is most frequent in the database. If the agent uses a similarity function that puts all its weight on the most recent outcome, her prediction will simply be that outcome.

Finally, *rule-based* reasoning calls for the agent to base her predictions on regularities that she believes characterize the phenomenon in question.

The boundaries between the three modes of reasoning are not always sharp. Our focus is on the Bayesian approach. By "Bayesian reasoning" we refer to the common approach in economic theory, according to which *all* reasoning is Bayesian. *Any* source of uncertainty is modeled in the state space, and all reasoning about uncertainty takes the form of updating a prior probability via Bayes' rule.

We present a framework that unifies these three modes of reasoning (and potentially others), allowing us to view them as special cases of a general learning process. The agent attaches weights to conjectures. Each conjecture is a set of states of the world, capturing a way of thinking about how outcomes in the world will develop. The associated weights capture the relative influence that the agent attaches to the various conjectures. The weighted sum of these conjectures is a Belief Function as in Dempster (1967) and Shafer (1976).

Given a sequence of observations, the agents rules out the conjectures that have been refuted by them, and continues with the weighted sum of the remaining ones. This turns out to be equivalent to Dempster-Shafer rule of combination, or updating of a belief function.

To generate a prediction, the agent sums the weight of all nontrivial conjectures consistent with each possible outcome, and then ranks outcomes according to their associated total weights. In the special case where each conjecture consists of a single state of the world, our framework is the standard Bayesian model, and the learning algorithm is equivalent to Bayesian updating. Employing other conjectures, which include more than a single state each, we can capture other modes of reasoning, as illustrated by simple examples of case-based and of rule-based reasoning.

Our model could be used to address either positive or normative questions. We focus on positive ones, describing how the reasoning process of an agent evolves as observations are gathered. Within the class of such questions, our model could be used to capture a variety of psychological biases and errors, but the focus of this paper is on the reasoning of an agent who makes no obvious errors in her reasoning. Such an agent may well be surprised by circumstances that she has deemed unlikely, that is, by "black swans," but will never be surprised by a careful analysis of her own reasoning. The optimality of this reasoning process is a normative question, which we do not address here.

Our main results concern the dynamics of the weight put on Bayesian vs. non-Bayesian reasoning. We suggest conditions under which Bayesian reasoning will give way to other modes of reasoning, and alternative conditions under which the opposite conclusion holds. Importantly, if the agent does not know the type of

[1] The talk is based on joint work with (i) Larry Samuelson and David Schmeidler (2013); (ii) Gabrielle Gayer (2014); (iii) Alfredo Di Tillio and Larry Samuelson (2013).

process she is facing, and attempts to be open-minded about it, Bayesian reasoning will disappear in the limit. The very simple reason is that there are many Bayesian conjectures, whereas other families of conjectures may be small. Specifically, the weight put on the Bayesian conjectures (as a whole) has to be divided among exponentially many disjoint subset, whereas the case-based ones (as well as some families of rule-based ones) are only polynomially large.

In a similar vein, we can also ask how the relative weight of rule-based and case-based conjectures changes with evidence. If a "rule" has to provide a prediction at each and every node, and be computable, we find that (i) if reality is simple enough (say, computable), then rule-based reasoning takes over; (ii) if reality isn't simple enough, then case-based reasoning is likely to be dominant.

Finally, the model can also be used to reason about counterfactuals.

References

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