De Finetti Coherence and Beyond

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The aim of the tutorial is to present the concept of coherence, which dates back to de Finetti, showing its role in managing incomplete (or missing) information.

We will start recalling the notion of coherence for (unconditional) probabilities and the related fundamental theorem.

Then, in order to generalize this notion to assessments on a set of conditional events, the axiomatic definition of conditional probability, essentially due to Renyi, de Finetti and Dubins, needs to be recalled together with the representation theorem of a conditional probability by means of a linearly ordered class of finitely additive measures.

Both for the unconditional and conditional case, de Finetti's coherence has a betting scheme interpretation and it can also be characterized in terms of solvability of a sequence of linear systems for each finite subset of conditional events.

One of the main peculiarities of de Finetti's coherence is that a coherent assessment can always be extended, generally not in a unique way, to any superset of (conditional) events, giving rise to a class of coherent extensions.

The relationship of coherence with the first fundamental theorem of the asset pricing will be underlined.

The role of coherence is particularly meaningful in Bayesian statistics where the extensions of a likelihood function and a prior probability need to be found. Even in this case the coherent extensions are not necessarily unique, and the whole class of coherent extensions needs to be considered. This leads to study lower and upper envelopes.

However, the coherent extensions could be required to satisfy some further properties such as disintegrability and conglomerability: this leads to distinguish different subclasses of extensions.

Models able to handle uncertainty in a more flexible way have favored the emergence of theories more general than classical probability.

The resulting uncertainty calculi, such as possibility measures, belief functions and k-monotone Choquet capacities, can be interpreted in terms of envelopes of de Finetti's coherent probabilities, also referred to as imprecise probabilities.

The main features of de Finetti's coherence are discussed in connection with its "generalizations" to imprecise probabilities, essentially given by Williams and Walley.

The coherence criteria given by Williams and Walley for imprecise probabilities differ in the way they face conditioning, so a comparison of the different notions will be presented.

Finally, the different notions of coherence for (conditional) random quantities will be reviewed by comparing Williams and Walley theories.

Some examples coming from applications will be used to illustrate key concepts.