A robust Bayesian analysis of the impact of policy decisions on crop rotations

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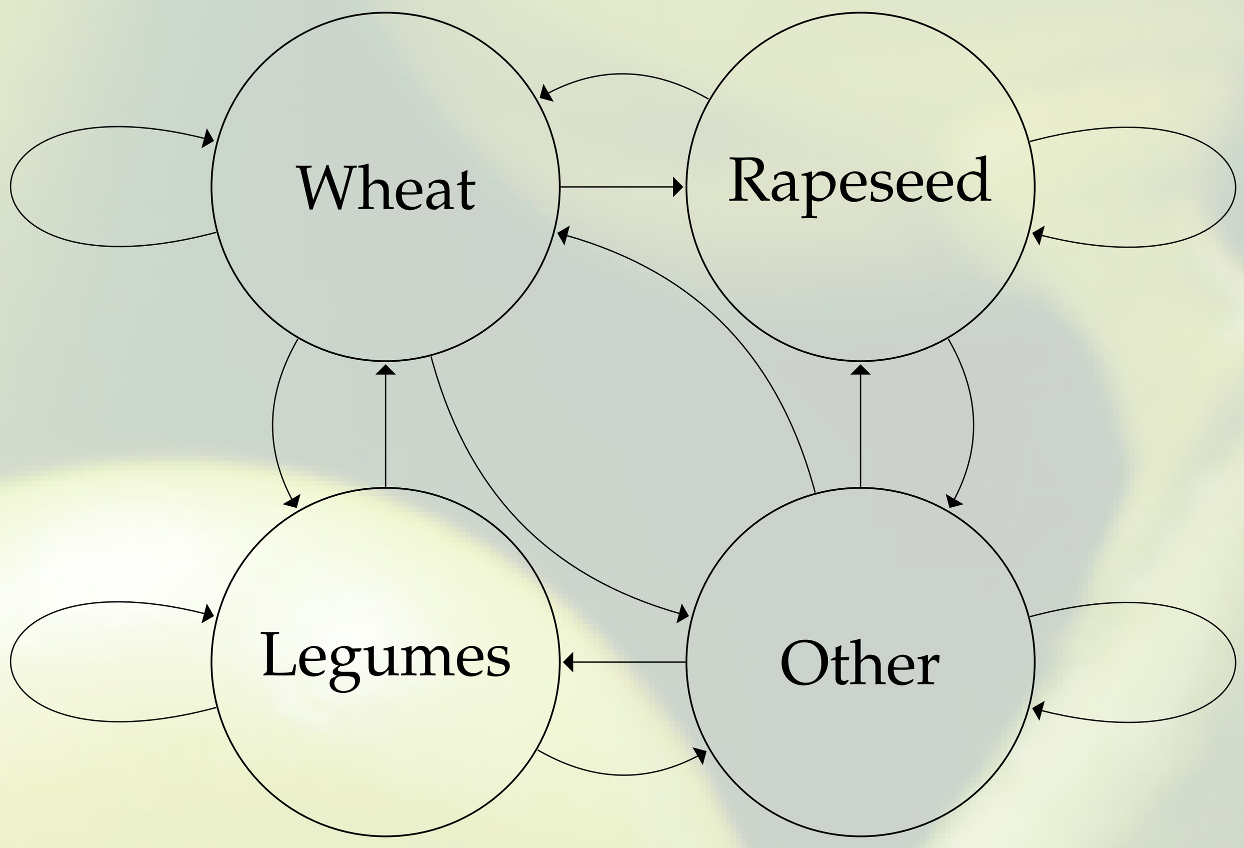
Introduction

- We investigate how we can use imprecise probability to inform policy.
- Land use model based on imprecise stochastic process.
- Imprecise due to scarce data, lack of expert opinion.
- Investigate how nitrogen price affects amount of legumes (peas and beans) being planted – hypothetical yet realistic scenario.

The Model



- Model transition probabilities as functions of rainfall and nitrogen price.
- Assume multinomial logit model for the probabilities, and define a conjugate prior distribution for the model parameters [1].
- Use broad class of priors to perform a robust Bayesian analysis – similar to the Imprecise Dirichlet Model [2].
- Computational complexity – use MAP estimate of posterior parameters [1].
- Calculate lower and upper posterior transition probabilities, and use these for inference [1].

Model validation

- Use cross validation to validate model.
- Single accuracy [3] – accuracy of predictions where only one crop is predicted: ~ 75%.
- Set accuracy [3] – accuracy of predictions when a set of crops is predicted: ~ 85%.
- Performs particularly well in regions of sparse data – shows benefits of using imprecise model.

[1] Lewis Paton et al. : Multinomial logistic regression on Markov chains for crop rotation modelling, IPMU, 2014
[2] Peter Walley: Inferences from Multinomial Data, Journal of the Royal Statistical Society, 58, 1996
[3] Giorgio Corani and Marco Zaffalon: Learning reliable classifiers from small or incomplete data sets: The naive credal classifier 2, Journal of Machine Learning Research, 9(4), 2008

Policy example

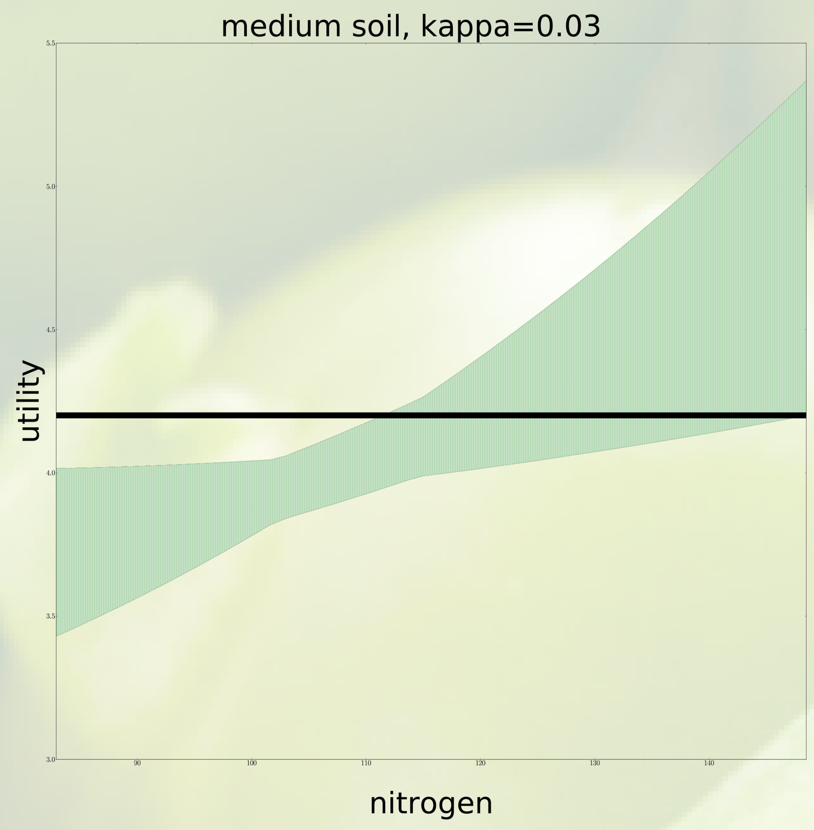
- Increased global meat demand so increased demand for protein crops e.g legumes [4].
- Higher nitrogen price ⇒ more legumes being grown.
- Aid policy maker to decide optimal nitrogen price.
- Assign simple linear utility function:

$$U(a, b, \kappa) = 100a - \kappa b$$

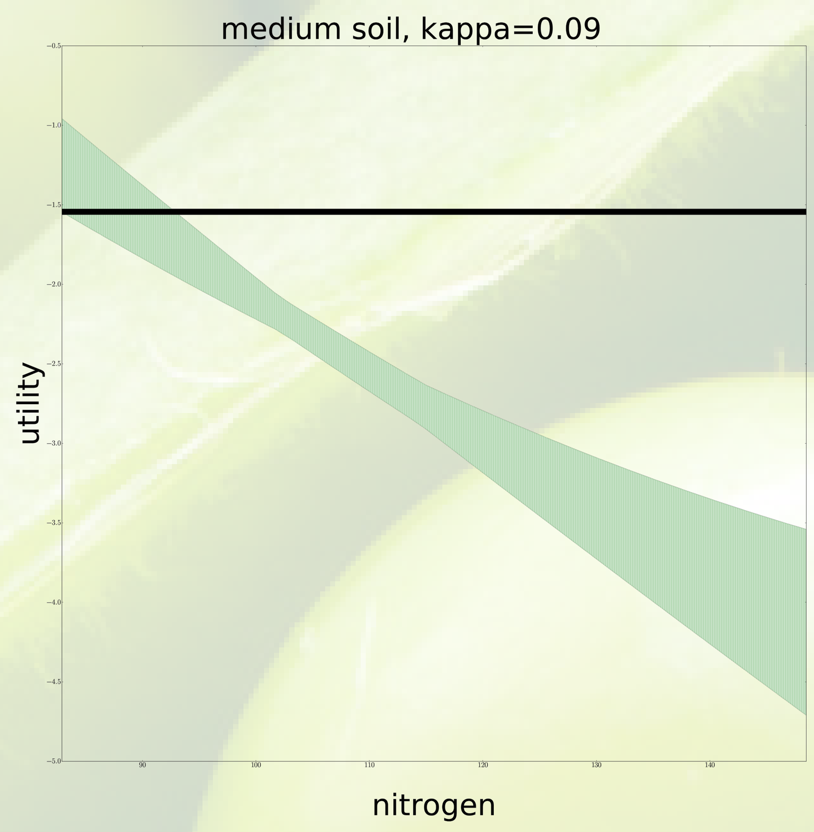
where:
  - a is the fraction of legumes grown
  - b is the nitrogen price
  - κ is a constant set by the decision maker
- Imprecise land use model results in lower and upper utilities: $\underline{U}(a, b, \kappa)$ and $\overline{U}(a, b, \kappa)$.
- Feasible set of b values $\mathcal{B}$.
- Use interval dominance [5]: all $b \in \mathcal{B}$ that satisfy

$$\overline{U}(b) \geqslant \max_{b \in \mathcal{B}} \underline{U}(b)$$

- Green region: $[\underline{U}(b), \overline{U}(b)]$
- Horizontal black line: $\max_{b \in \mathcal{B}} \underline{U}(b)$
- Low κ:



- Higher nitrogen price preferred. Policy decision – higher tax on nitrogen.
- High κ:



- Lower nitrogen price preferred. Policy decision – lower tax on nitrogen.

Conclusions

- Project shows benefits that imprecise probability can bring to real world problems – aid policy makers.
- Model validation suggest the model performs well.

[4]: C Gilbert and C Morgan: Food price volatility, Philosophical Transactions of the Royal Society of London, 365, 2010
[5]: Matthias C. M. Troffaes: Decision making under uncertainty using imprecise probabilities, IJAR, 45, 2007