Using imprecise continuous time Markov chains for assessing the reliability of power networks with common cause failure and non-immediate repair

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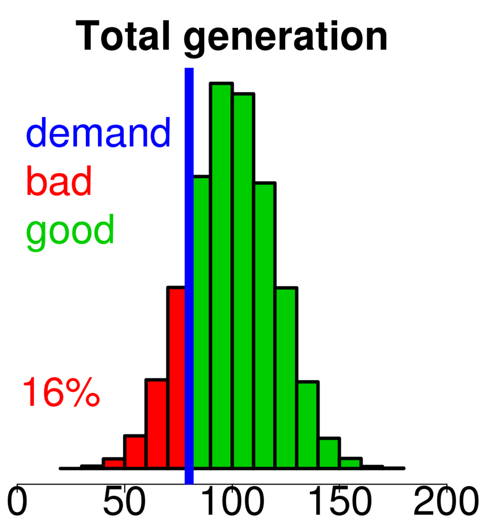
Motivating Example

- random components, say power generators with capacity X_i

- deterministic known risk threshold, say power demand x

Risk Analysis

$$\textit{risk} = \Pr\left(\sum_{i=1}^{n} X_i \le x\right)$$



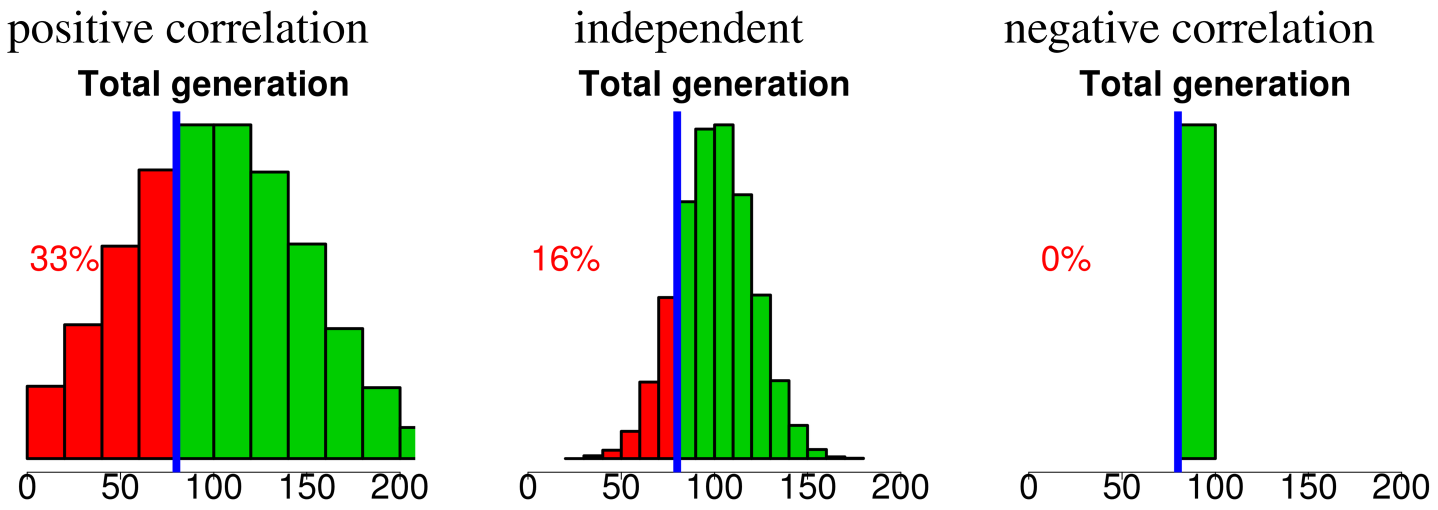
in common cases:

- threshold well x known (not always!)
- distribution of $\sum_{i=1}^{n} X_i$ is *very sensitive to modelling assumptions*

Modelling Assumptions On $\sum_{i=1}^{n} X_i$?

- *marginal* distributions of each X_i = easy to get right
- *interactions* between the different X_i = easy to get wrong

positive correlation | independent | negative correlation



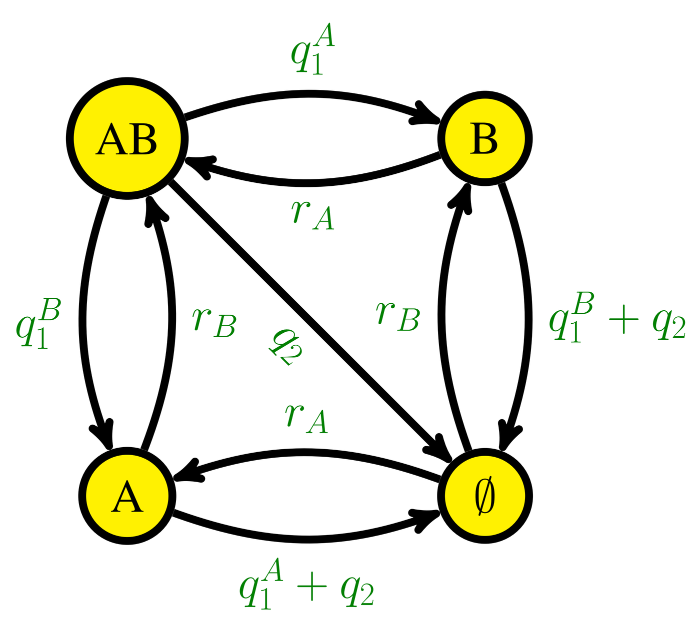
= typical reality — why? *common cause events*

= typical assumption — why? *computational convenience*

= unusual

Risk Model from Network Operator Perspective

- *How frequently do outages occur?*
- *How long do outages last for?*



Issues

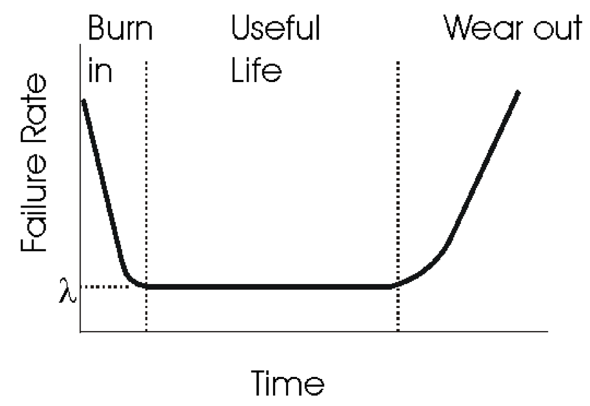
- Markov assumption violated in reality.
- Little data to estimate parameters.

We can use imprecision to address both issues.

Issues With the Precise Markov Chain Model

Continuous time Markov chains have nice computational properties, but…

Failure rates not constant in time

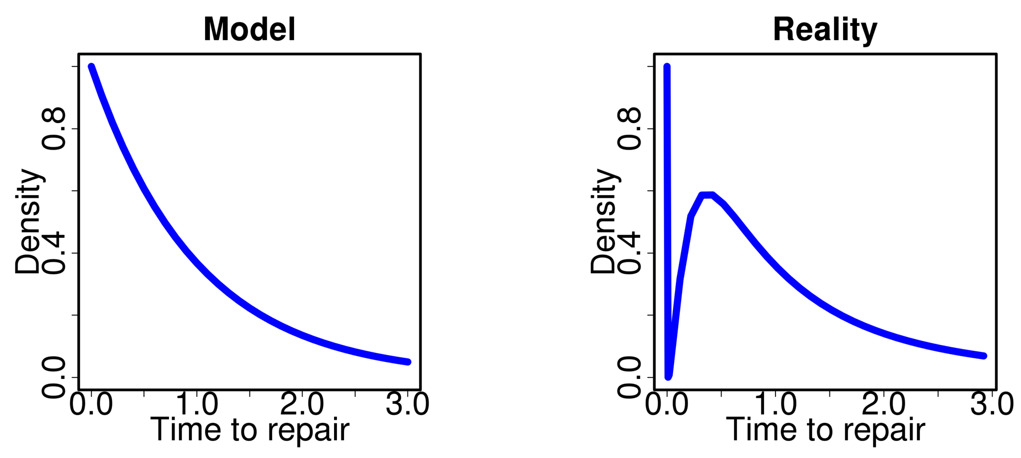


Epistemic uncertainty about rates themselves

Particularly for common cause events!

Violation of Markov condition

Repair rates depend on system history, and not exponentially distributed:



Missing covariates

Rates depend on operation of the entire power system.

Imprecision To The Rescue!!

Continuous Time Imprecise Markov Chains

Relax transition rates so they:

- **can depend on time**.
- **can depend on history**

but the bounds on the rates cannot!

Practical Considerations

- *efficiently* compute inferential bounds? yes, subject to a technical condition
- still produce *useful* bounds on inferences? yes!

$$\underline{E}(f(X_t) \mid X_0 = i) = \lim_{n\to\infty} \left[\left(I + (t/n)\underline{Q}\right)^n f\right]_i \quad (1)$$

$$\underline{\pi}_i := \lim_{t\to\infty} \underline{P}(X_t = i \mid X_0 = j) = \lim_{t\to\infty} \lim_{n\to\infty} \left[\left(I + (t/n)\underline{Q}\right)^n I_i\right]_j \quad (2)$$

Theorem For any period of length τ far into the future:

$$[\tau\underline{\pi}_i, \tau\overline{\pi}_i] \quad \text{expected time spent in state } i \quad (3)$$

$$\left[\tau\sum_{j\neq i}\underline{\pi}_j[\underline{Q}I_i]_j, \tau\sum_{j\neq i}\overline{\pi}_j[\overline{Q}I_i]_j\right] \quad \text{expected number of transitions to state } i \quad (4)$$

Conclusions

Novel mathematics like imprecise Markov chains enable a *much wider class of statistical processes* reducing model discrepancy and improving risk analysis. But…

- How to get model parameter bounds from data in general?
- Improved algorithms for "imprecise matrix exponential".
- Additional covariates: failure rates depend on system context.

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