Common Knowledge, Ambiguity, and the Value of Information in Games

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Introduction & Main Results

In an individual decision problem, a Bayesian decision maker should not refuse to postpone her terminal decision in order to acquire cost-free information, thereby implying a non-negative value of information to the individual. This paper asks whether this result holds in the context of games.

1. In Bayesian games, players may assign negative value to cost-free information.
2. The non-negative value of information can be restored by relaxing the assumption of common knowledge (CK) in Bayesian games.
3. In games under ambiguity, players may also assign negative value to cost-free information.
4. Nevertheless, when ambiguity is present, the conclusion about the negative value of information in games is robust with respect to the weakening of the common knowledge assumption.

Bayesian Game 1a

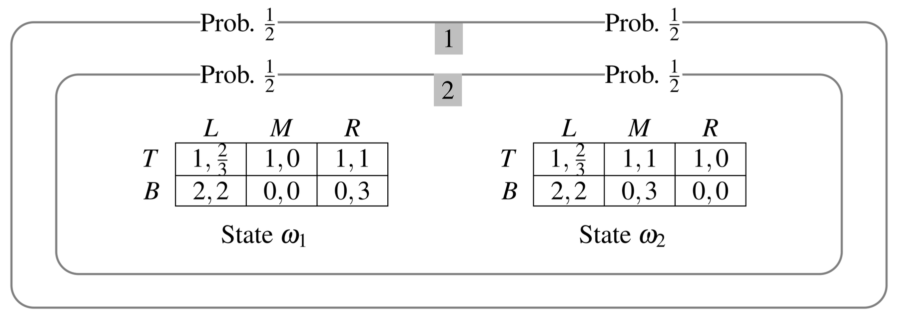

Figure 1: No information about the state

- By expectation, player 2's strategy L strictly dominates the other strategies.
- (B, L) is the unique Bayesian Nash equilibrium (BNE) for Game 1a: $(2, 2)$.

Bayesian Game 1b

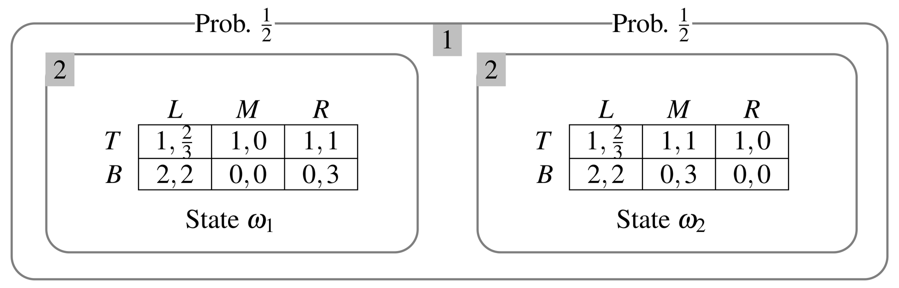

Figure 2: Player 2 knows the state

- Each type of player 2 has a strictly dominant strategy, R and M respectively.
- $(T, (R, M))$ is the unique Bayesian Nash equilibrium for Game 1b: $(1, (1, 1))$.

Negative Value of Information in Bayesian Games

Suppose that player 2 is first asked to choose either to play Game 1a or to play Game 1b, and then enters the chosen game. Which game does player 2 prefer to play?

- The unique BNE of Game 1a yields player 2 a higher expectation than the one of Game 1b.
- The initial choice is to play Game 1a, even though player 2 is informed of the state in Game 1b.
- So, player 2 has negative value for the information about the state.

Game 1c: Relax CK Assumption

Consider another version of Game 1a: Player 2 knows the state, but player 1 falsely believes that player 2 assigns prob. $\frac{1}{2}$ to both states.

- Given player 1's false belief, she chooses B as the best response.
- Given player 2's information, her optimal choice is to play (R, M).
- The solution $(B, (R, M))$ for Game 1c yields player 2 the expectation of 3.

Game 1a vs. Game 1c: Positive Value

Consider a similar sequential problem for player 2. What is the initial choice for player 2 between Game 1a and Game 1c?

- Note that player 2 strictly prefers to play Game 1c with more information.
- By contrast, player 2 has positive value for the information.
- Reason: By relaxing the CK assumption, **act-state dependence** is no longer present.

Game 2a under Ambiguity

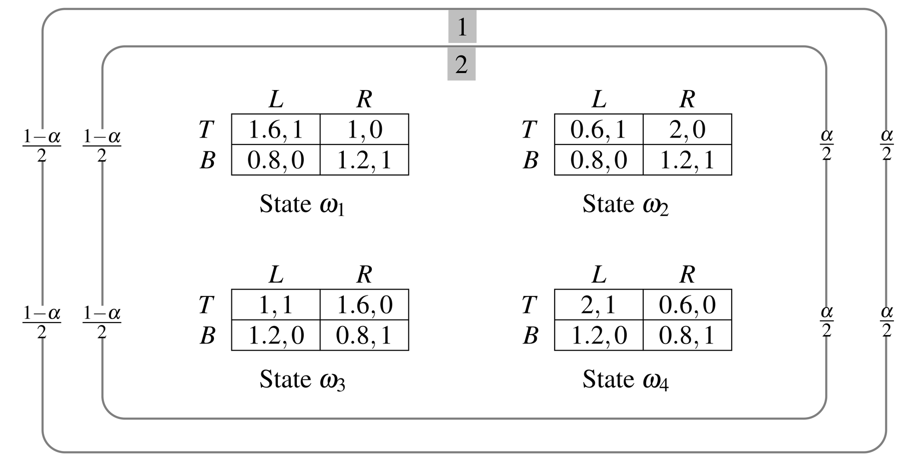

Figure 4: Game under ambiguity & $\alpha \in [0.1, 0.9]$

	L	R
T	1.3, 1	1.3, 0
B	1, 0	1, 1

Figure 5: Game 2a in strategic form

Game 2b under Ambiguity

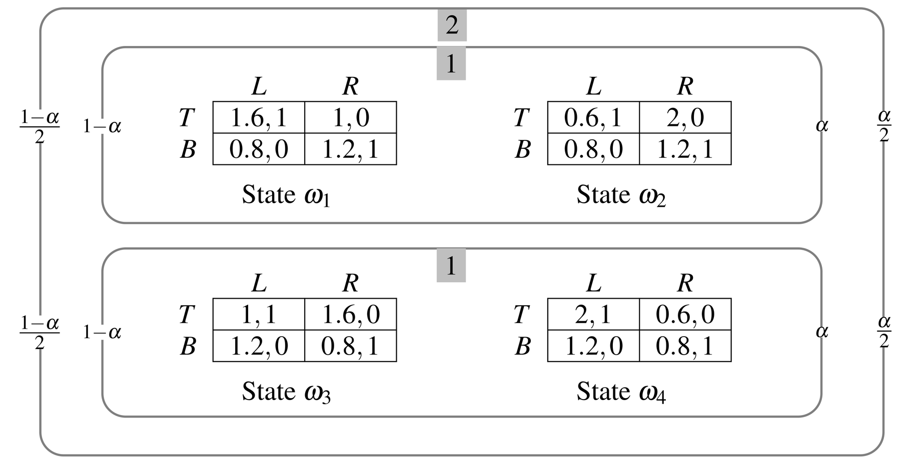

Figure 3: Player 1 has more information

- According to Γ-maximin, player 1 should choose (B, B) no matter whether player 2 plays L or R.
- Player 1's best response is to play R.
- Thus, $((B, B), R)$ is the unique Γ-maximin equilibrium for Game 2b: $((1.2, 0.8), 1)$.

Negative Value of Information in Games under Ambiguity

Consider the following sequential problem for player 1: First, choose between Game 2a and 2b, and then play that game. Which game does player 1 prefer to play?

- Similarly, the solution to Game 2a yields player 1 a higher expectation than the one of Game 2b.
- The initial choice is to play Game 2a, even though player 1 has more information in Game 2b.
- So, player 1 has negative value for the information about the state.

Game 2c: Relax CK Assumption

Consider a modified version of Game 2a: Player 1 obtains more information about the state, but player 2 is not informed of this fact.

- Given player 2's false belief, she chooses L as the best response.
- Given player 1's information, her optimal choice is to play (B, B).
- The unique solution $((B, B), L)$ for Game 2c yields the outcome $((0.8, 1.2), 0)$.

Game 2a vs. 2c: Negative Value

Consider a similar sequential problem for player 1. What is the initial choice for player 1 between Game 2a and Game 2c?

- Player 1 still strictly prefers to play Game 2a with less information, thereby implying a negative value of information.
- **Dilation** occurs: the prior prob. interval for event $\{\omega_1, \omega_4\}$ is strictly contained within its conditional prob. interval.

Concluding Remarks

- The role act-state dependence plays in the sequential setting is fundamental to understanding the result of negative value of information in Bayesian games: Probabilistic dependence between a player's initial choice and her prob. about opponents' strategy choices.
- In games under ambiguity, players may have negative value for new information, even if act-state dependence is no longer present by relaxing the CK assumption. This is mainly due to the phenomenon of dilation of sets of probabilities.