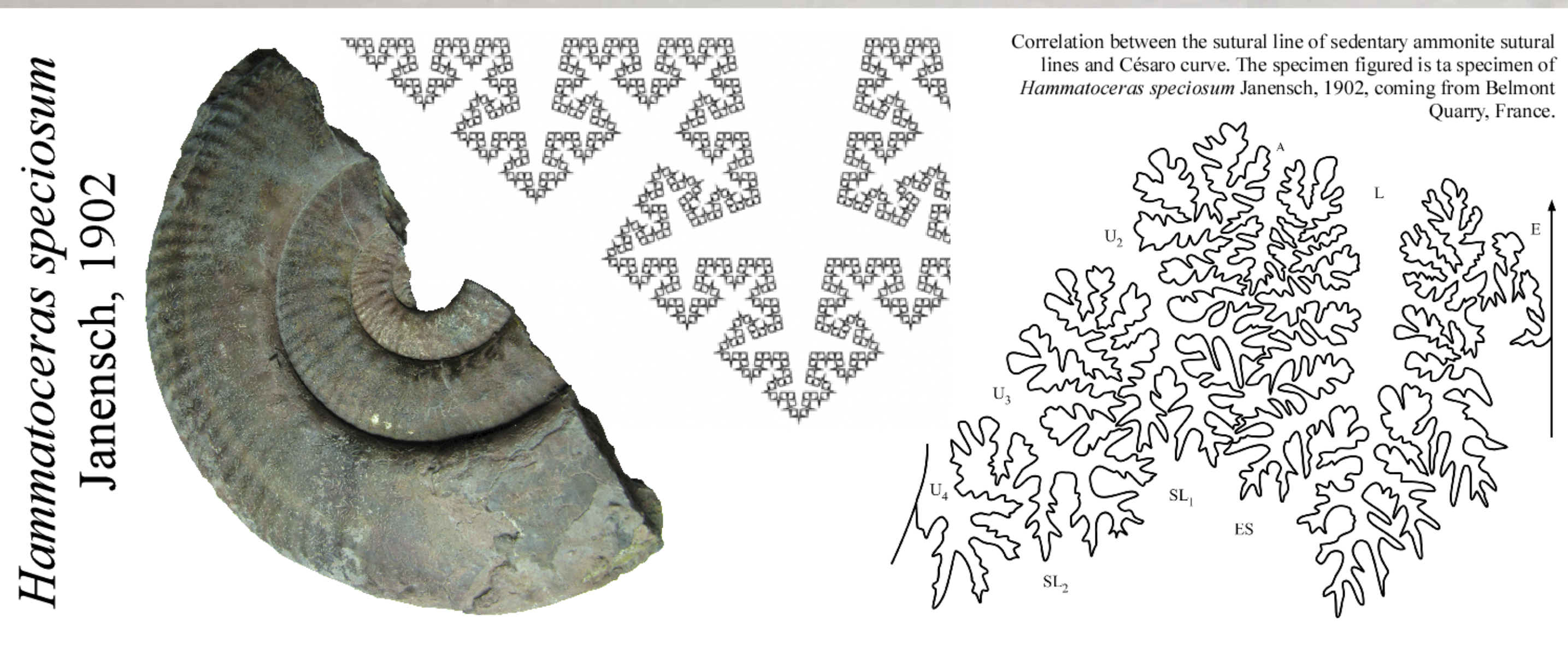
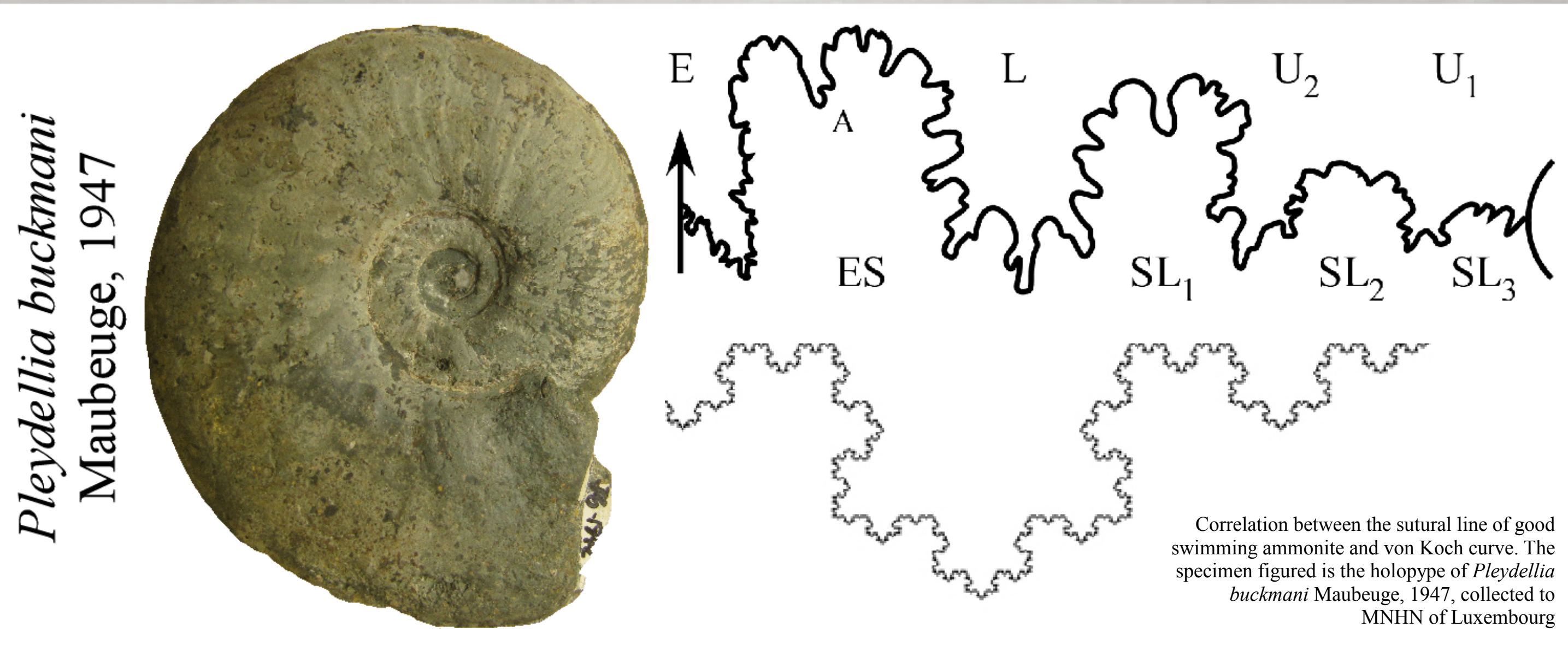
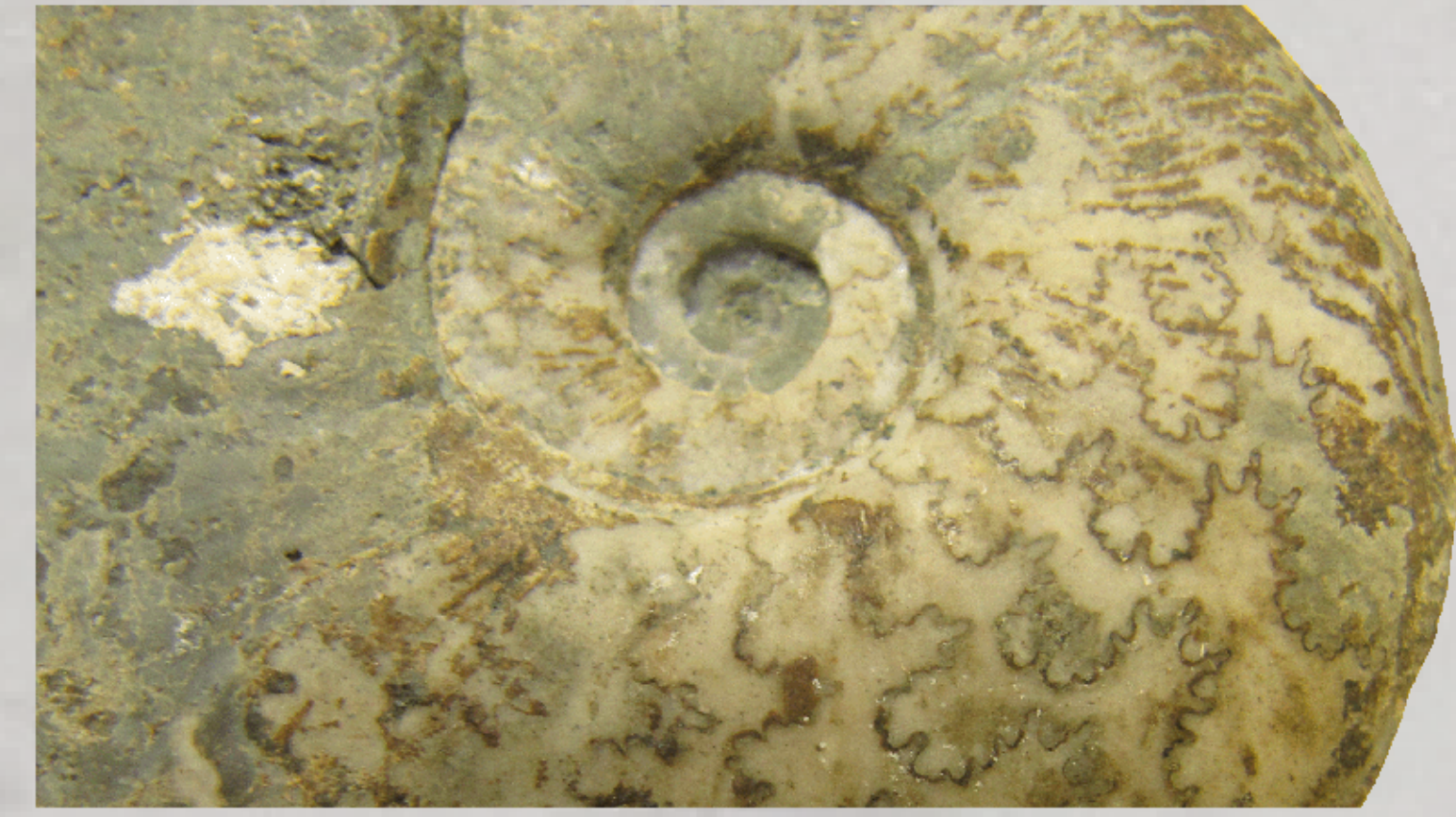


Ammonites are extinct ectococled molluscs belonging to the Class Cephalopoda which lived during the Mesozoic Era. Their usefulness in Jurassic and Cretaceous paleontology and dating of rocky bodies (biostratigraphy) is widely proved. For this reason, they are studied by several authors worldwide in order to achieve information regarding their habitats and climate of past world. Coherent upper conditional previsions defined with respect to Hausdorff outer measures are used to make a probabilistic analysis of the paleo-environmental causes that generated complex sutural lines. In particular, the role of hydrostatic pressure is studied.

The shell of ammonites is sub-divisible in three parts: protoconch, phragmocone and body chamber. The protoconch is the first chamber of cephalopod, developed during the larval ontogenetic stage. The phragmocone is the chambered portion of shell, between the protoconch and the body chamber. Chambers are separated, each other, by septa. The geometric projection of septum on the inner side of the shell is the sutural line. Every sutural line is characterized by alternation of several elements, named saddles and lobes, which reflect a fractal geometrical development. During geological time, sutural lines become very and very complicated, developing more indented elements, probably in answer to conquest, during evolution, of new habits. In any cases some kind of sub-division of rocks, based on complexity of sutural lines is possible. In parallel with the ontogenetic development of specimen, the sutural lines become more and more indented increasing number of elements and their complexity. The spaces between septa are filled, in the living shell, by a mixing of water and air. Its pressure, governed by cephalopod, permits to animal to move in vertical direction in the column of water. The sutural lines are often overlapped on gerontic stages, giving probable life troubles. The portion of shell between the last septa and the opening is the body chamber, the place where the animal lived and from where it did the life activities. The length of body chamber varies between one third and more than one whorl. There is some connection between length of body chamber and complexity of sutural line.



The sutural lines of ammonites may be considered as fossilized adaptive answer to paleo-environmental command, and its study become very important in order to reconstruct the life habits of this important group of fossils. Sutural lines of Toarcian (lower Jurassic) ammonites are made of almost two separated groups. The first is close to mathematical model of the von Koch curve, not very complicated ones, characterized by several, well developed elements and one example is the sutural line of *Pleydellia buckmani* Maubeuge 1947, on table. This kind of sutural line is connected to compressed, acute, keeled, hydrodynamic shell, ornamented by sinuous ribs. The body chamber is short and the position of molluscs is below the shell with the opening backward. This is the arrangement of a good swimmer ammonites which may cover high distances during their movement. The second group of sutural lines is close to mathematical model of the Césaro curve. This kind of sutural lines has very complicated elements that are narrow and very indented. One example is *Hammatoceras speciosum* Janensch, 1902, figured on table. They are associated to sub-circular in shape and generally non-hydrodynamic shells, with little or absent keels, spines in external or inner portion, rectiradiate, primary and/or secondary ribs. The body chamber is very long, sometimes over than one whorls, the position of barycenter is changed and the opening is moved upward. This arrangement does not permit to have good swimming skills. These are the features of so-called sedentary ammonites, which prefer to move up and down in the column of water. In order to study the paleo-environmental causes of the complexity of the sutural lines, we interpret ammonites as complex systems whose evolution during time is described by a finite family of contractions; the attractor of this family represents the sutural line, whose complexity is measured in terms of Hausdorff dimension.



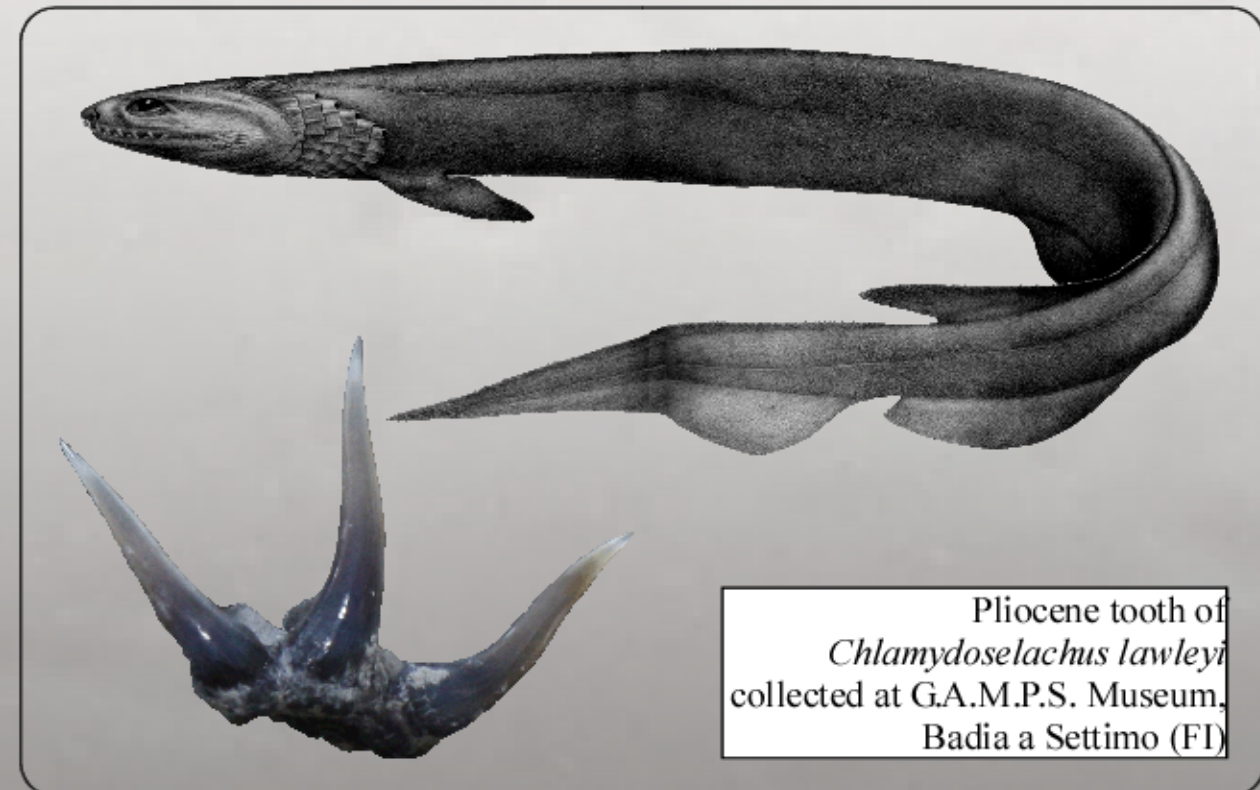
The hydrostatic pressure is represented by a random variable $X(\omega)$ defined on Ω and we calculate the Choquet integral of this random variable given the sutural line, which is the conditioning event B . We consider three cases.

First Case - Constant pressure

The hydrostatic pressure is constant, $X(\omega) = m$; in that case we have that

$$\bar{P}(X|B) = \frac{1}{h^s(B)} \int_B m dh^s = m$$

This implies that the upper prevision that an ammonite suffered constant pressure is greater as the ammonite lived in depth.



Examples are the good swimmer ammonites, as *Pleydellia buckmani* described and figured above, which do not need change their depth of life, thanks to its capacity to move horizontally (point 1 in the figure). Ammonites are completely extinct animal and ours are only conjecture, but actually exists several other species of animal which live constantly to a predeterminate depth at constant pressure, as, for example the abyssal frilled shark *Chlamydoselachus*. Its fossilized teeth permit to recognize abyssal sediments, in Pliocene successions.

Third case - Irregular pressure

Let consider a pressure $Y(\omega)$ and a the sutural line B with non-integer Hausdorff dimension equal to s , ($\dim_H(B) = s$). If all the weak upper level sets

$$\{\omega \in B : Y(\omega) \geq x\} \text{ con } x \in \mathbb{R}$$

have Hausdorff dimension less than s thus $\bar{P}(Y|B) = 0$.

We can say that $\bar{P}(Y|B)$ is different from zero if the ammonite underwent the pressure in subsets of B with Hausdorff dimension equal to B . In that case the function $Y(\omega)$ should be very irregular and this fact could be a confirmation of the total freedom of movement of the ammonite during the rise to the surface. If the hydrostatic pressure is described by a very irregular function $Y(\omega)$ it would correspond to repeated and sudden movements in vertical of the ammonite.

Examples are the non good swimmer ammonites, as *Hammatoceras speciosum* described and figured above (point 3 in the figure). These ammonites show all those features to indicate a life habits connected with high depth, in places where predators are more rare than in others. Their other life activities as feed, reproduction and sociality must be done close to surface. This means almost daily up and down migration.

Second Case - Strictly decreasing pressure

The hydrostatic pressure is strictly decreasing; this situation describes a different life style. In fact an ammonite that from high depth surface rooms underwent strictly decreasing pressure. We fix $Y(\omega) = -\omega + m$. We suppose that the sutural line, represented by the set B is regular with Hausdorff dimension equal to 1 and we compare $\bar{P}(X|B)$ and $\bar{P}(Y|B)$.

We have that

$$\bar{P}(X|B) = \frac{1}{h^1(B)} \int_0^m h^1 \{\omega \in B : X(\omega) \geq x\} dx = m$$

and

$$\begin{aligned} \bar{P}(Y|B) &= \frac{1}{h^1(B)} \int_0^m h^1 \{\omega \in B : Y(\omega) \geq x\} dx = \\ &= \frac{1}{h^1(B)} \int_0^m (-x + m) dx = \frac{1}{h^1(B)} \left[-\frac{1}{2}x^2 + mx \right]_0^m = \frac{1}{h^1(B)} \frac{1}{2}m^2 \end{aligned}$$

so that

$$\text{if } 0 < m < 2h^1(B) \text{ we have } \bar{P}(Y|B) < \bar{P}(X|B)$$

Thus for low values of the pressure and regular sutural line is likely that the pressure has been constant.

Otherwise

$$\text{if } m > 2h^1(B) \text{ we have } \bar{P}(Y|B) > \bar{P}(X|B)$$

that is it is more likely for ammonites with with regular sutural line (Hausdorff dimension equal to 1) that the pressure on the points of the suture line has been a decreasing function with maximum value equal to m . This should happen for ammonites that lived very deep and that to survive they had to rise to the surface to feed.

Examples are connected, for the first sub-case, to abandon of nursery during ontogenetic growth of ammonites. The young ammonites show very simple sutural lines which become more and more complex during growth: This features suggests a migration forward the place of life, (spot 1 in figure). The nursery of ammonites may be placed in protected areas, perhaps in shallow and clear waters, where the probability of surviving is very high, far from dangers of open sea.

Similarly for the case 3, examples for the second sub-case are linked to sedentary ammonites as described *Hammatoceras speciosum*. This group of ammonites, living protected in deep water, must migrate upward, close to surface, for the life activities as feed, reproduction and sociality.

The results show that the Choquet integral of the hydrostatic pressure given the sutural line is a mathematical tool to describe different life styles of ammonites, which determined the complexity in the sutural lines.

