
Viking Sound and Light

Using imprecise continuous time Markov chains for assessing the reliability of power networks with common cause failure and non-immediate repair

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joint work with
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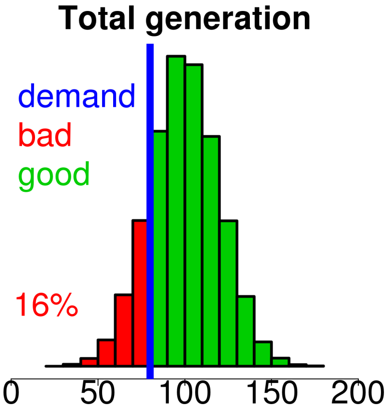
Motivating Example

- random components, say power generators with capacity X_i

- deterministic known risk threshold, say power demand x

Motivating Example

$$\text{risk} = \Pr\left(\sum_{i=1}^{n} X_i \leq x\right)$$



in common cases:

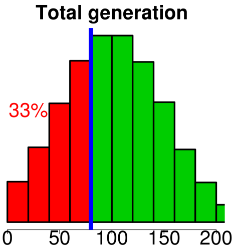
- threshold well x known (not always!)
- distribution of $\sum_{i=1}^{n} X_i$ is very sensitive to modelling assumptions

Motivating Example

Modelling Assumptions On $\sum_{i=1}^{n} X_i$?

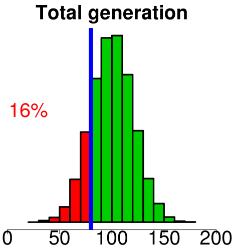
- marginal distributions of each X_i = easy to get right
- interactions between the different X_i = easy to get wrong

positive correlation



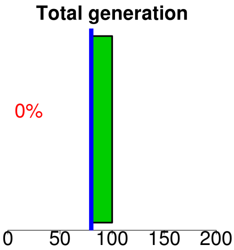
= typical reality
why?
common cause events

independent



= typical assumption
why?
computational convenience

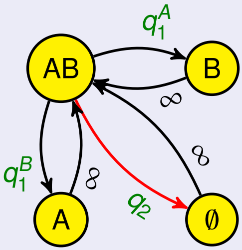
negative correlation



= unusual

The Instant Repair Model

Continuous Time Markov Chain



model parameters:

- q_2: common-cause failure rate
- q_1^A & q_1^B: 'single-cause' failure rate per component

Markov assumption:

- failure rates unaffected by history

Parameterisation: Alpha-Factor Model

$$q_2 = \frac{\alpha_2}{\alpha_1 + 2\alpha_2}(q_t^A + q_t^B) \quad (1)$$

$$q_1^A = q_t^A - q_2 \quad (2)$$

$$q_1^B = q_t^B - q_2 \quad (3)$$

'directly observable' parameters:

- α_2 = fraction of faults due to common cause $\quad \alpha_1 = 1 - \alpha_2$
- q_t^A & q_t^B = failure rates of components seen separately

The Data

Expert Knowledge

- α_2: directly from nationwide statistics
- q_t^A & q_t^B: from nationwide statistics about constituents e.g. from
  - average failure rate per km of overhead line
  - average failure rate per km of underground cable
  - any other components ...

regional dependencies?

how applicable is nationwide data to a specific network?

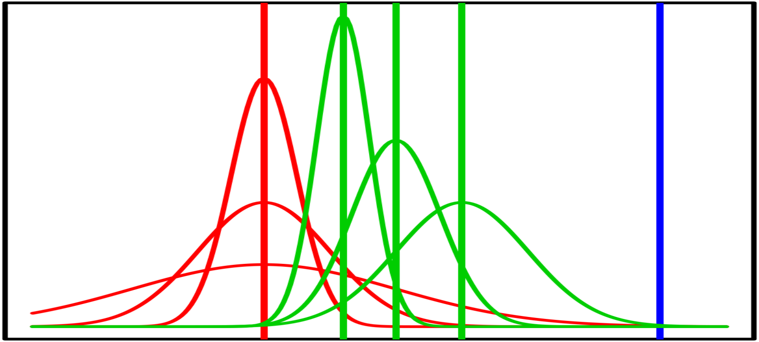
Direct Failure Data about the Actual Network Under Study

typically, very sparse, but more informative

we propose imprecise probability to cope with lack of relevant data and uncertainty about applicability of expert opinion

The Analysis: Why Imprecise Probability?

- nationwide statistics inform the mean of the prior
- we consider a set of variances for the prior
- this is an elegant way to handle prior-data conflict [13]

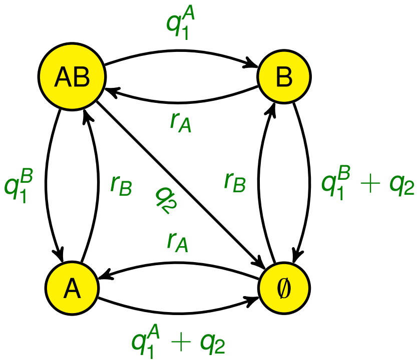


variance of prior = judgement about how fast you are willing to learn from data

Incorporating Repair: Why?

What do network operators care about?

- How frequently do outages occur?
  Approximately answered by previous analysis even though instant repair assumption is not realistic.
- How long do outages last for?
  Need explicit repair model! How? Billinton & Allan [1]:



Issues

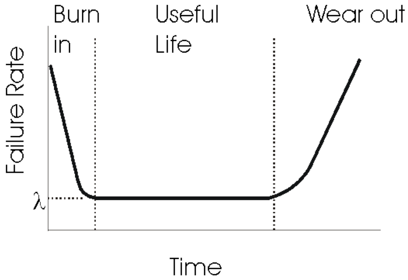
- Markov assumption violated in reality.
- Little data to estimate parameters.

We can use imprecision to address both issues.

Incorporating Repair: Issues With The Standard Model

Model assumptions blatantly violated in even the simplest systems!

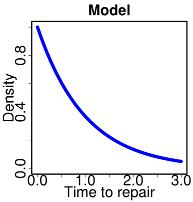
- violation of stationarity
  Failure rates are usually not constant in time: bathtub curve.

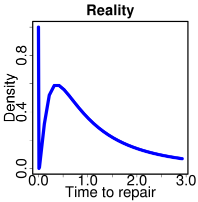


- epistemic uncertainty
  There can be substantial uncertainty about the rates themselves, particularly for common cause events.

Incorporating Repair: Issues With The Standard Model

- violation of Markov condition
  Repair rates depend on system history, and repair times are not exponentially distributed as predicted by the model.





- missing covariates
  Repair rates depend on operation of the entire power system. For instance: severe wheather, many simultaneous failures, but number of repair crews is limited.

. . . so, why use this model?

Incorporating Repair: Why Use The Standard Model?

Computation!

$$E(f(X_t) \mid X_0 = i) = [e^{tQ} f]_i \tag{4}$$

- e^{tQ} = matrix exponential
- many efficient methods for calculating it (although numerical stability sometimes an issue)

Other examples:

$\pi Q = 0$	long term distribution π	(5)
$\tau\pi_i$	expected time spent in state i	(6)
$-\tau\pi_i Q_{ii}$	expected number of transitions to state i	(7)

Imprecise Continuous Time Markov Chains

Concerns

- Can we efficiently compute inferential bounds?

  Yes, subject to a technical condition
  (but can always do so conservatively).

- Can this very large class of statistical processes still produce useful bounds on inferences?

  Apparently, yes!

Theorem (Škulj, 2012)

Under fairly relaxed technical conditions:

$$\underline{\mathrm{E}}(f(X_t) \mid X_0 = i) = \lim_{n \to \infty} \left[\left(I + (t/n)\underline{Q} \right)^n f \right]_i \tag{8}$$

where $[\underline{Q}f]_i := \min_{Q \in \mathcal{Q}} [Qf]_i$

Conclusion

- Sparse data and prior-data conflict? Bayes + sensitivity analysis.
- Relaxing Markov assumption and stationarity via bounding?
  It is easy and inferences still sufficiently precise.
- Novel mathematics like imprecise Markov chains enable
  a much wider class of statistical processes
  reducing model discrepancies and improving risk analysis.
- Imprecision not only useful for epistemic uncertainty,
  also useful to study very complex models in an efficient way.

Future work:

- How to get model parameter bounds from data in general?
- Improved algorithms for “imprecise matrix exponential”.
- Additional covariates: failure rates depend on system context.
- Decision making for power network design:
  tradeoff between cost of redundancy and common-cause risks.
- Potential for applications to other practical problems.

Thank you for listening & visit the poster!

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