Common Knowledge, Ambiguity, and the Value of Information in Games

Hailin Liu

Department of Philosophy
Carnegie Mellon University

ISIPTA '15
20 July 2015, Pescara, Italy

Main Results

- Value of information in Bayesian games:
  - Negative value of information with common knowledge (Osborne, 2004).
  - Positive value of information without common knowledge.
- Value of information in incomplete information games under ambiguity:
  - Negative value of information with common knowledge.
  - Negative value of information even without common knowledge.

Osborne's Example: Negative Value

- Case 1a: Both players do not know the state.

State ω_1 (probability $\frac{1}{2}$):

	L	M	R
T	$1, {}^2/_3$	$1, 0$	$1, 1$
B	$2, 2$	$0, 0$	$0, 3$

State ω_2 (probability $\frac{1}{2}$):

	L	M	R
T	$1, {}^2/_3$	$1, 1$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

BNE

(B, L)

$(2, 2)$

Osborne's Example: Negative Value

- Case 1a: Both players do not know the state.

State ω_1 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 0$	$1, 1$
B	$2, 2$	$0, 0$	$0, 3$

State ω_2 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 1$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

BNE

(B, L)

$(2, 2)$

- Case 1b: Player 2 knows the state.

State ω_1 (probability 1):

	L	M	R
T	$1, 2/3$	$1, 0$	$1, 1$
B	$2, 2$	$0, 0$	$0, 3$

State ω_2 (probability 1):

	L	M	R
T	$1, 2/3$	$1, 1$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

BNE

$(T, (R, M))$

$(1, (1, 1))$

Relax Common Knowledge Assumption

- Case 1a: No information about the state.

State ω_1 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 0$	$1, 1$
B	$2, 2$	$0, 0$	$0, 3$

State ω_2 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 1$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

BNE

(B, L)

$(2, 2)$

Relax Common Knowledge Assumption

- Case 1a: No information about the state.

State ω_1 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 0$	$1, 1$
B	$2, 2$	$0, 0$	$0, 3$

State ω_2 (probability $\frac{1}{2}$):

	L	M	R
T	$1, 2/3$	$1, 1$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

BNE

(B, L)

$(2, 2)$

- Case 1c: Without common knowledge.
  - Reasoning based on available information: Expected utility maximization.
  - Player 2 knows the state: Strict dominance gives rise to (R, M).
  - Player 1 is uncertain about the state, and *falsely* believes that player 2 is uncertain about the state: B.
  - Solution: $(B, (R, M))$ with $(0, (3, 3))$.

Example: Game under Ambiguity

- 2 players, 2 actions for each player, and 4 possible states.
- The players' ambiguity about the state: A common set of priors

$$\mathcal{P} = \{p \in \Delta(\Omega) : p(\omega_1) = p(\omega_3) = \tfrac{1-\epsilon}{2}, p(\omega_2) = p(\omega_4) = \tfrac{\epsilon}{2}, \epsilon \in [0.1, 0.9]\}.$$

- Player 1's signal function: $\tau_1(\omega_1) = \tau_1(\omega_2) = t_1$, $\tau_1(\omega_3) = \tau_1(\omega_4) = t_1'$.
- Player 2's signal function: $\tau_2(\omega_k) = t_2$ for $k = 1, 2, 3, 4$.

	ω_1 ($\frac{1-\epsilon}{2}$): L	ω_1: R	ω_2 ($\frac{\epsilon}{2}$): L	ω_2: R	ω_3 ($\frac{1-\epsilon}{2}$): L	ω_3: R	ω_4 ($\frac{\epsilon}{2}$): L	ω_4: R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1

Example: Negative Value and Ambiguity

- Case 2a: Both players do not receive any signals.

	$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$		$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$	
	L	R	L	R	L	R	L	R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1
	ω_1		ω_2		ω_3		ω_4	

- A unique (interim) Γ-maximin equilibrium (T, L): $(1.3, 1)$.

Example: Negative Value and Ambiguity

- Case 2a: Both players do not receive any signals.

	$\frac{1-\epsilon}{2}$			$\frac{\epsilon}{2}$		$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$	
	L	R		L	R	L	R	L	R
T	1.6, 1	1, 0		0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1		0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1
	ω_1			ω_2		ω_3		ω_4	

- A unique (interim) Γ-maximin equilibrium (T, L): $(1.3, 1)$.
- Case 2b: Player 1 receives two signals.

	$1-\epsilon$				ϵ			
	L	R	L	R	L	R	L	R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1
	ω_1		ω_2		ω_3		ω_4	

- A unique interim Γ-maximin equilibrium $((B, B), R)$: $((1.2, 0.8), 1)$.

Another Comparison without Common Knowledge

- Case 2a: Both players do not receive any signals.

	$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$		$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$	
	L	R	L	R	L	R	L	R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1
	ω_1		ω_2		ω_3		ω_4	

- A unique (interim) Γ-maximin equilibrium (T, L): $(1.3, 1)$.

Another Comparison without Common Knowledge

- Case 2a: Both players do not receive any signals.

	ω_1: $\frac{1-\epsilon}{2}$		ω_2: $\frac{\epsilon}{2}$		ω_3: $\frac{1-\epsilon}{2}$		ω_4: $\frac{\epsilon}{2}$	
	L	R	L	R	L	R	L	R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1

- A unique (interim) Γ-maximin equilibrium (T, L): $(1.3, 1)$.
- Case 2c: Player 1 has more information but player 2 doesn't know that.
  - Reasoning based on available information: Γ-maximin.
  - Player 1 receives two signals: B always yields a higher lower expectation.
  - Player 2 has false beliefs: Choose L as a best response to T.

Another Comparison without Common Knowledge

- Case 2a: Both players do not receive any signals.

	$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$		$\frac{1-\epsilon}{2}$		$\frac{\epsilon}{2}$	
	L	R	L	R	L	R	L	R
T	1.6, 1	1, 0	0.6, 1	2, 0	1, 1	1.6, 0	2, 1	0.6, 0
B	0.8, 0	1.2, 1	0.8, 0	1.2, 1	1.2, 0	0.8, 1	1.2, 0	0.8, 1
	ω_1		ω_2		ω_3		ω_4	

- A unique (interim) Γ-maximin equilibrium (T, L): $(1.3, 1)$.
- Case 2c: Player 1 has more information but player 2 doesn't know that.
  - Reasoning based on available information: Γ-maximin.
  - Player 1 receives two signals: B always yields a higher lower expectation.
  - Player 2 has false beliefs: Choose L as a best response to T.
- Negative value of information: In Case 2c, $((B, B), L)$ results in $((0.8, 1.2), 0)$; player 1 is still worse off compared to Case 2a.

Summary: Comparisons

- Contrast concerning the value of information in games.

	Bayesian Games	Games under Ambiguity
Common Knowledge	Negative	Negative
No Common Knowledge	Positive	Negative

- To account for the differences:
  - Act-state dependence occurs in Bayesian games.
  - Dilation occurs in incomplete information games under ambiguity.