How to choose among choice functions

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Core idea

We are interested in rational decision making where the agent is represented as having a set of probability functions $\mathcal{P}$ for her degrees of beliefs, or credences. The main question is: what can we say about rational choices?

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We describe some popular "choice functions", explore what properties they have, and whether these properties are rationally compelling. We also explore the question of how to interpret the choice function.

Basics

- We have a (finite) state space Ω.
- We have a set of gambles Φ which are functions from Ω to $\mathbb{R}$.
- For $p \in \mathbb{R}$ let $p\varphi + (1-p)\psi$ be the "mixed act" defined pointwise, and let $p\varphi + (1-p)A$ be the set of acts in A mixed with φ.
- A^* is the set of mixed acts over acts in A.
- The agent has a set of probability functions $\mathcal{P}$ from Ω to $[0,1]$.
- Expectations for $\mathbf{pr} \in \mathcal{P}$ defined by $E_p(\varphi) = \sum_{\omega \in \Omega} \mathbf{pr}(\omega)\varphi(\omega)$.
- Expectations for imprecise agents: $\mathcal{E}(\varphi) = \{\mathrm{E}_{\mathbf{pr}}(\varphi), \mathbf{pr} \in \mathcal{P}\}$.
- Summary functions $\overline{\mathcal{P}}(X) = \inf \mathcal{P}(X)$ and $\underline{\mathcal{P}}(X) = \sup \mathcal{P}(X)$ likewise for $\mathcal{E}$.
- For $A \subseteq \Phi$ let $\mathcal{C}(A)$ be the set of choiceworthy acts.

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Very Weak All we can say is that the best act is among the $\varphi \in \mathcal{C}(A)$.

Maximality

The *maximal set* for a relation $\succeq$ is $\mathcal{M}_{\succeq}$:

$$\mathcal{M}_{\succeq}(A) = \{\varphi \in A : \neg\exists\psi \in A, \psi \succ \varphi\}$$

Expectation relations

We define the following two relations:

$$\varphi \succeq_{\mathrm{E}_{\mathsf{pr}}} \psi \text{ iff } \mathrm{E}_{\mathsf{pr}}(\varphi) \geq \mathrm{E}_{\mathsf{pr}}(\psi)$$

$$\succeq_{\mathrm{Dom}} = \bigcap_{\mathcal{P}} \succeq_{\mathrm{E}_{\mathsf{pr}}}$$

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E-admissibility $L(A) = \bigcup_{\mathbf{pr} \in \mathcal{P}} \mathcal{M}_{\succeq_{\mathrm{E}_{\mathbf{pr}}}}(A)$

Properties of choice

Nondomination $\mathcal{C}$ satisfies $\succeq_{\mathrm{Dom}}$.

Contraction Consistency $\mathcal{C}(A \cup B) \subseteq \mathcal{C}(A) \cup \mathcal{C}(B)$. (Sen's alpha)

Independence $\mathcal{C}(pA + (1-p)\varphi) = p\,\mathcal{C}(A)(1-p)\varphi$

Union Consistency $\mathcal{C}(A) \cap \mathcal{C}(B) \subseteq \mathcal{C}(A \cup B)$. (Sen's gamma)

All-or-Nothing If $\varphi \in \mathcal{C}(A)$ but $\varphi \notin \mathcal{C}(B)$ then, for all $\psi \in \mathcal{C}(A)$ we have $\psi \notin \mathcal{C}(B)$. (Sen's beta)

Mixing $\mathcal{C}(A) \subseteq \mathcal{C}(A^*)$.

Convexity $\mathcal{C}(A)^* \cap A = \mathcal{C}(A)$.

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- One can compose E-admissibility and Maxmin with Maximality to avoid nondomination, but all the other problems remain.

Further work

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- A “regret-based” rule?